CTNT 2022, JUNE 9 - 11
SCHEDULE, TITLES, AND ABSTRACTS

TITLES AND ABSTRACTS (Alphabetical by last name)

Eran Assaf (Dartmouth College)

Title: Definite orthogonal modular forms in rank 4
Abstract: We consider spaces of modular forms attached to positive definite quadratic forms in four variables, and make explicit their connection to Hilbert modular forms using the even Clifford functor. By relating it to the theory of theta lifts we obtain an explicit theta correspondence, and gain a full understanding of the systems of Hecke eigenvalues that can arise. We will discuss further applications to non-vanishing and Eisenstein congruences, and touch upon the picture in higher rank.

This is joint work with Dan Fretwell, Colin Ingalls, Adam Logan, Spencer Secord and John Voight.
Mode: In Person

Lea Beneish (UC Berkeley)

Title: Degrees of Points on Curves
Abstract: Given a plane curve C defined over $\mathbb{Q}$, when the genus of the curve is greater than one, FaltingsвЂ™ theorem tells us that the set of rational points on the curve is finite. It is then natural to consider higher degree points, that is, points on this curve defined over fields of degree d over $\mathbb{Q}$. We ask for which natural numbers d are there points on the curve in a field of degree d. We will discuss this question first for superelliptic curves, giving geometric heuristics relating to the scarcity or abundance of various degrees of points. In more generality, there is a lot of structure in the set of values d, some of which we will explain in this talk. Parts of this talk are based on joint work with Andrew Granville and parts of the talk are based on joint work with Christopher Keyes.
Mode: in person

Alexander Betts (Harvard)

Title: Grothendieck's section set and the Lawrence–Venkatesh method
Abstract: In 1983, shortly after Faltings' resolution of the Mordell Conjecture, Grothendieck formulated a number of conjectures to the effect that the Diophantine geometry of hyperbolic curves over number fields should be controlled by their fundamental groups. Most famously, his Section Conjecture posits that the set of rational points on a projective curve Y of genus at least two should be equal to the set of splittings of a certain exact sequence of fundamental groups. However, to date precious little is known unconditionally about these sets of splittings: for example, is the number of splittings even finite?

In this talk, we will discuss some recent work with Jakob Stix, in which we prove some partial finiteness results in this direction, valid for all Y. The key new idea is to apply methods developed

by Brian Lawrence and Akshay Venkatesh in their new proof of the Mordell Conjecture to the set of splittings of the fundamental exact sequence.
Mode: in person

Edgar Costa (MIT)

Title: Geometric invariants from counting points
Abstract: In this talk, we will focus on how one can deduce some geometric invariants of an abelian variety or a K3 surface by studying their Frobenius polynomials, i.e., counting points. In the case of an abelian variety, we show how to obtain the decomposition of the endomorphism algebra, the corresponding dimensions, and centers. Similarly, by studying the variation of the geometric Picard rank, we obtain a sufficient criterion for the existence of infinitely many rational curves on a K3 surface of even geometric Picard rank.
Mode: in person

Jack Dalton (University of South Carolina)

Title: Extreme Covering Systems
Abstract: A covering system is a finite set of congruences such that all integers lie in at least one of the congruence classes. A covering system is called distinct if all the moduli are different. Erdős asked if the smallest modulus in a distinct covering system could be arbitrarily large. The minimum modulus problem, as this question became known, was resolved by Bob Hough when he proved that the minimum modulus of a distinct covering system must be smaller than 10^50. This has since been improved to 616,000. We ask the related question: If the smallest modulus of a covering system is m, what is the smallest that the largest modulus can be? We will show that for all $m > 2$, there does not exist a distinct covering system with all of the moduli in the interval $[m, 8m]$. This is joint work with Ognian Trifonov.
Mode: (in person)

Vefa Goksel (University of Massachusetts-Amherst)

Title: Misiurewicz polynomials and dynamical units
Abstract: We study the dynamics of the unicritical polynomial family $f_{d,c}(z) = z^d + c \in \mathbb{C}[z]$. The c-values for which $f_{d,c}$ has a strictly preperiodic postcritical orbit are called *Misiurewicz parameters*, and they are the roots of *Misiurewicz polynomials.* The arithmetic properties of these special parameters have found applications in both arithmetic and complex dynamics. In our recent work, we investigate some new such properties. In particular, we consider the algebraic integers obtained by evaluating a Misiurewicz polynomial at a different Misiurewicz parameter, and we ask when these algebraic integers are algebraic units. This question naturally arises from some results recently proven by several authors, and it is also evocative of the study of dynamical units introduced by Morton and Silverman. We propose a conjectural answer to this question, which we prove in many cases. This is a joint work with Rob Benedetto.
Mode: In person

Xiaoyu Huang (CUNY Graduate Center)

Title: On the universal deformation ring of a residual Galois representation with three Jordan Holder factors and R = T theorems.
Abstract: In this work, we study Fontaine-Laffaille, essentially self-dual deformations of a mod p non-semisimple Galois representation with its Jordan Holder factors being three mutually non-isomorphic absolutely irreducible representations. We established sufficient conditions for the universal deformation ring to be a discrete valuation ring. We also get an R = T theorem given a sufficient bound on a quotient of Hecke algebra. Potential applications to abelian surfaces with rational cycles and Ikeda lifts will be discussed.
Mode: In Person

Tyler Genao (University of Georgia)

Title: Typically bounding torsion on elliptic curves isogenous to rational j-invariant
Abstract: In 1996, Merel proved the well-known "strong uniform boundedness conjecture" for elliptic curves: for each integer $d > 0$, there exists a constant $B(d)$ such that if F is any number field of degree d, then for all elliptic curves $E_{/F}$ one has $\#E(F)[\text{tors}] < B(d)$. Unfortunately, explicit upper bounds $B(d)$ on torsion subgroups over degree $d > 3$ number fields are greater than exponential in d. Despite this, special families of elliptic curves, such as the family of elliptic curves with complex multiplication, have torsion subgroups which satisfy a "typical order": there exists a bound on this family's torsion subgroups which works over *all* degrees, provided one ignores a subset of degrees of arbitrarily small upper density. Such families of elliptic curves are said to be *typically bounded in torsion.*

In this talk, we will introduce the family $\mathcal{I}_F$ of elliptic curves isogenous to an F-rational j-invariant. We will motivate a study of its elements' torsion subgroups, and see some of the key ideas in showing that it's typically bounded in torsion. We will also see how such a family fits into a more general study of torsion subgroups of $\mathbb{Q}$-curves.
Mode: In-person

Mathilde Gerbelli-Gauthier (Institute for Advanced Study)

Title: Growth of cohomology of Picard modular surfaces: an illustrated example of Langlands functoriality
Abstract: How fast do Betti numbers grow in a congruence tower of covering spaces? IвҐЄll discuss this question in the special case of Picard modular surfaces, which are 4-dimensional real manifolds. There, the question is most interesting in degree 1, for which there is expected to be very little cohomology. I'll explain how the problem is related to automorphic forms, and specifically how the dearth of cohomology classes is a consequence of Langlands functoriality.
Mode: In Person

Akash Jena (Indiana University)

Translation functors for locally analytic representations:

Abstract: Let G be a p-adic Lie group with reductive Lie algebra $\mathfrak{g}$. In analogy to the translation functors introduced by Bernstein and Gelfand on categories of $U(\mathfrak{g})$-modules we consider similarly defined functors on the category of modules over the locally analytic distribution algebra $D(G)$ on which the center of $U(\mathfrak{g})$ acts locally finite. These functors induce equivalences between certain subcategories of the latter category. Furthermore, these translation functors are naturally related to those on category $\mathcal{O}$ via the functors from category $\mathcal{O}$ to the category of coadmissible modules. As an application, we investigate the effect of the translation functors on locally analytic representations $\Pi(V)^{\mathrm{la}}$ associated by the p-adic Langlands correspondence for $GL_2(\mathbb{Q}_p)$ to 2-dimensional Galois representations V.
Mode: In-person

Christopher Keyes (Emory University)

Title: Local solubility in families of superelliptic curves
Abstract: If one chooses at random an integral binary form $f(x, z)$ of fixed degree d, what is the probability that the superelliptic curve with equation $C\colon : y^m = f(x, z)$ has a p-adic point, or better, points everywhere locally? In joint work with Lea Beneish, we show that the proportion of forms $f(x, z)$ for which C is everywhere locally soluble is positive, given by a product of local densities. By studying these local densities, we produce bounds which are suitable enough to pass to the large d limit. In the specific case of curves of the form $y^3 = f(x, z)$ for a binary form of degree 6, we determine the probability of everywhere local solubility to be 96.94%, with the exact value given by an explicit infinite product of rational function expressions.
Mode: In Person

Seoyoung Kim (Queen's University)

Title: On Markoff type surfaces over number fields and the arithmetic of Markoff numbers
Abstract: In this talk, we introduce some interesting arithmetic properties of solutions to inhomogeneous Markoff-Hurwitz equations, in particular we study counterexamples to the Hasse principle as well as the question of finite generation of the solution set under Vieta involutions, sign changes and permutations of coordinates. This is a joint work with D. Schindler and J. Sivaraman.
Mode: Online

Avi Kulkarni (Dartmouth College)

Title: The p-adic integral geometry formula
Abstract: The integral geometry formula relates the volume of a real projective hypersurface to the expected number of real intersections with a random line. By random, we mean with respect to a probability measure invariant under the action of the rotation group SO(n+1) on real projective n-space. In this talk, I will discuss the p-adic analogue of this formula. Joint work with Antonio Lerario.
Mode: In Person

Colette LaPointe (CUNY Graduate Center)

Title: Some dynatomic modular curves in positive characteristic
Abstract: In arithmetic dynamics, dynatomic modular curves like $Y_1(n)$ and $Y_0(n)$ have frequently been studied for the family $f_c(x) = x^d + c$, but less is known for other one-dimensional polynomial families, especially in positive characteristic. We are studying when $Y_1(n)$ and $Y_0(n)$ have good reduction modulo p for the families $f_c(x) = x^p \pm x + c$. In the current research we have shown that over characteristic p, the curve $Y_1(n)$ for the family $f_c(x) = x^p + x + c$ is smooth if and only if $p \nmid n$, except in the case $n = p = 2$ where $Y_1(n)$ is smooth. $Y_1(n)$ has also been shown to be non-reduced when $n = p^k$ for $k \geq 1$ (except in the case $n = p = 2$), and from computation on Sage, we know $Y_1(n)$ can be reducible even in some cases when $Y_1(n)$ is smooth. Some further questions now being explored include finding when $Y_1(n)$ irreducible, and what can be said on the smoothness or irreducibility of $Y_0(n)$. This is work for my dissertation under the direction of Andrew Obus.
Mode: In-person

Sung Min Lee (University of Illinois at Chicago)

Title: On the Acyclicity of $\tilde{E}_p(\mathbb{F}_p)$ for Primes in Arithmetic Progressions
Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$. Let us denote the reduction of E modulo a prime p by $\tilde{E}_p$. Recently, Akbal and G üloğlu studied on the cyclicity of $\tilde{E}_p(\mathbb{F}_p)$ for p in arithmetic progression. In this talk, we study the issue of which arithmetic progressions a mod n have the property that, for all but finitely many primes $p \equiv a$ mod n, the group $\tilde{E}_p(\mathbb{F}_p)$ is NOT cyclic. This is a joint work with Nathan Jones.
Mode: In Person

Diana Mocanu (University of Warwick)

Title: The Modular Approach to solving Diophantine Equations over totally real fields.
Abstract: In this talk I will give a brief overview of the modular approach for solving Diophantine equations over the rationals pioneered by Wiles, Ribet, and Mazur in solving FermatвЂ™s Last Theorem. I will show how it generalizes over totally real fields to give asymptotic results by studying the solutions of certain S-unit equations. If time permits, I will present a few examples of recent results involving this method.
Mode: Online

Jackson Morrow (UC Berkeley)

Title: Boundedness of Hyperbolic Varieties
Abstract: Let C_1, C_2 be smooth projective curves over an algebraically closed field K of characteristic zero. What is the behavior of the set of non-constant maps $C_1 \to C_2$? Is it infinite, finite, or empty? It turns out that the answer to this question is determined by an invariant of curves called the genus. In particular, if C_2 has genus $g(C_2) \geq 2$ (i.e., C_2 is hyperbolic), then there are only finitely many non-constant morphisms $C_1 \to C_2$ where C_1 is any curve, and moreover, the degree of any map $C_1 \to C_2$ is bounded linearly in $g(C_2)$.

In this talk, I will explain the above story and discuss a higher dimensional generalization of this result. To this end, I will describe the conjectures of Demailly and Lang which predict a relationship between the geometry of varieties, topological properties of Hom-schemes, and the behavior of rational points on varieties. To conclude, I will sketch a proof of a variant of these conjectures, which roughly says that if X/K is a hyperbolic variety, then for every smooth projective curve C/K of genus $g(C) \geq 0$, the degree of any map $C \to X$ is bounded uniformly in $g(C)$.
Mode: in person

Tung T. Nguyen (Western University)

Title: On the arithmetic of generalized Fekete polynomials
Abstract: Fekete polynomials play an essential role in studying special values of L-functions of quadratic fields. In previous work, we have examined the arithmetic of Fekete polynomials associated with a Dirichlet character with a prime conductor. In this talk, we will introduce the generalized Fekete polynomial associated with an arbitrary Dirichlet character. We then determine their cyclotomic and non-cyclotomic factors. Furthermore, we will show that the Galois group of these generalized Fekete polynomials seems to follow a rather elegant pattern.
Mode: Online

Oana Padurariu (Boston University)

Title: Rational points on hyperelliptic Atkin-Lehner quotients of modular curves
Abstract: Let $X_0(N)^*$ be the quotient of the modular curve $X_0(N)$ by the full group of Atkin-Lehner involutions. In this talk, I will discuss the computation (using various Chabauty techniques) of rational points on $X_0(N)^*$, when the modular star quotient is hyperelliptic. This is joint work with Nikola Adzaga, Shiva Chidambaram, and Timo Keller.
Mode: in person

Lillian Pierce (Duke University)

Title: On Superorthogonality
Abstract: The Burgess bound is a well-known upper bound for short multiplicative character sums, which implies for example a subconvexity bound for Dirichlet L-functions. Since the 1950's, people have tried to improve the Burgess method. In order to try to improve a method, it makes sense to understand the bigger "proofscape" in which a method fits. The Burgess method didn't seem to fit well into a bigger proofscape. In this talk we will show that in fact it can be regarded as an application of "superorthogonality." This perspective links topics from harmonic analysis and number theory, such as Khintchine's inequality, Walsh-Paley series, square function estimates and decoupling, multi-correlation sums of trace functions, and the Burgess method. We will survey these connections in an accessible way, with a focus on the number theoretic side.
Mode: Online

Ciaran Schembri (Dartmouth College)

Title: Reducing models for branched covers of the projective line
Abstract: We discuss a method for reducing models of curves equipped with a map to the projective line which is unramified away from three points. Using the ramified points of the map, we can compute "small" functions supported at these points to produce "small" plane models of the original curve. It is then possible to rescale a plane model in an optimal way to reduce the size of the coefficients, for which we use an integer linear program. We implemented and ran the algorithm on a database of Belyi maps in the LMFDB with often very favourable results. This is joint work with Sam Schiavone and John Voight.
Mode: in person

Yunqing Tang (Princeton University)

Title: The unbounded denominators conjecture
Abstract: The unbounded denominators conjecture, first raised by Atkin and Swinnerton-Dyer in 1968, asserts that a modular form for a finite index subgroup of $\mathrm{SL}_2(\mathbb{Z})$ whose Fourier coefficients have bounded denominators must be a modular form for some congruence subgroup. Our proof of this conjecture is based on a new arithmetic algebraization theorem, which has its root in the classical Borel-Dwork rationality criterion. In this talk, we will discuss some ingredients in the proof and a variant of our arithmetic algebraization theorem, which we will use to prove the irrationality of certain 2-adic zeta value. This is joint work with Frank Calegari and Vesselin Dimitrov.
Mode: Online