CTNT 2020, JUNE 12 - 14
SCHEDULE, TITLES, AND ABSTRACTS

CODE OF CONDUCT

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CONFERENCE SCHEDULE AT A GLANCE (Times are US/Eastern - UTC-4)

		DAY	
TIME	**Friday, June 12**	**Saturday, June 13**	**Sunday, June 14**
9:30 - 10	Informal greetings and chat		
10 - 10:05	Welcome	Announcements	Announcements
10:05 - 10:45	**PETE L. CLARK**	**RAVI RAMAKRISHNA**	**ASHER AUEL**
10:45 - 11:15	Break	Break	Break
11:15 - 11:55	**ALEXANDRA FLOREA**	**WANLIN LI**	**JEREMY ROUSE**
12 - 1:30	Lunch	**Group photo at noon!** and Lunch	Lunch
1:30 - 1:50	*Andrew Kobin*	*Jeffrey Yelton*	*David Corwin*
1:50 - 2:05	Break	Break	Break
2:05 - 2:45	**BIANCA VIRAY**	**JEREMY BOOHER**	**DAVID SAVITT**
2:45 - 3:00	Break	Break	Break
3:00 - 3:20	*Matilde Lalín*	*Rakvi*	*Debanjana Kundu*
3:20 - 3:35	Break	Break	Break
3:35 - 3:55	*Arvind Suresh*	*Holly Paige Chaos*	*Vaidehee Thatte*
3:55 - 4:10	Break	Break	Break
4:10 - 4:30	*Benjamin Breen*	*Garen Chiloyan*	**JAN VONK**
4:30 - 4:50	Break		
4:50 - 5:30	Panel Discussion		Closing remarks

TITLES AND ABSTRACTS (Alphabetical by last name)
PLENARY TALKS

ASHER AUEL (Dartmouth)

Title: Elliptic curves and the local-global principle for quadratic forms
Abstract: The Hasse–Minkowski theorem says that a quadratic form over a global field admits a nontrivial zero if it admits a nontrivial zero everywhere locally. Over more general fields of arithmetic and geometric interest, such as function fields of varieties over number fields, the failure of the local-global principle is often controlled by auxiliary structures of interest, such as torsion points of the Jacobian, elements of the Tate–Shafarevich group, and the Brauer group. I will explain work with V. Suresh on the failure of the local-global principle for quadratic forms over function fields of varieties of dimension at least two. We construct counterexamples that are controlled by higher unramified cohomology groups of certain Calabi–Yau varieties of generalized Kummer type that are associated to products of elliptic curves over the rational numbers. Along the way, we develop an arithmetic version of a result of Gabber on the nontriviality of unramified cohomology on products of elliptic curves, which leads to an amusing open problem where elliptic curves whose j-invariants are certain algebraic units cause the most difficulty.

JEREMY BOOHER (Canterbury)

Title: G-Valued Crystalline Deformation Rings in the Fontaine-Laffaille Range.
Abstract: This is joint work with Brandon Levin. Let G be a split connected reductive group over a p-adic field with residue field k. Fix a continuous homomorphism ρ from the absolute Galois group of an unramified extension of $\mathbf{Q}_p$ to $G(k)$. Understanding "nice" lifts of ρ to characteristic zero is an essential tool for studying Galois representations. We show that when the p-adic Hodge type μ is "Fontaine–Laffaille", the crystalline deformation ring for ρ with p-adic Hodge type μ is formally smooth over $W(k)$. We do so by relating the deformation ring to a smooth locus inside an affine Grassmanian using Kisin modules. For classical groups, this gives a new proof of formal smoothness independent of Fontaine–Laffaille theory.

PETE L. CLARK (UGA)

Title: CM Points on Modular Curves: Volcanoes and Reality.
Abstract: For an imaginary quadratic order $\mathcal{O}$ and a positive integer N, I have (mostly!) determined the number of closed $\mathcal{O}$-CM points on the modular curve $X_0(N)$ and their fields of moduli. One can deduce from this the number of closed $\mathcal{O}$-CM points on $X_1(N)$ and their degrees, recovering recent work with Abbey Bourdon. We proceed by an analysis of the action of complex conjugation on isogeny volcanoes. Isogeny volcanoes are mostly familiar to those who compute with elliptic curves over finite fields. In this talk I will introduce them from scratch and explain their use in the study of CM elliptic curves over number fields.

ALEXANDRA FLOREA (Columbia)

Title: Non-vanishing for cubic L-functions.
Abstract: Chowla conjectured that $L(1/2, \chi)$ never vanishes, for χ any Dirichlet character. Soundararajan showed that more than 87.5% of the values $L(1/2, \chi_d)$, for χ_d a quadratic character, do not vanish. Much less is known about cubic characters. Baier and Young showed that more than $X^{6/7-\epsilon}$ of $L(1/2, \chi)$ are non-vanishing, for χ a primitive, cubic character of conductor of size up to X. I will talk about recent joint work with C. David and M. Lalin, where we show that a positive proportion of these central L-values are non-vanishing in the function field setting. This is achieved by computing the first mollified moment using techniques previously developed by the authors in their work on the first moment of cubic L-functions, and by obtaining a sharp upper bound for the second mollified moment, building on work of Lester and Radziwill.

WANLIN LI (MIT)

Title: Ceresa class and hyperelliptic curves
Abstract: The Ceresa cycle is an algebraic cycle associated to a smooth pointed curve. It is algebraically trivial for hyperelliptic curves and non-trivial for a generic curve of genus ≥ 3 over the complex numbers. The Ceresa class is a Galois cohomology class derived from the Ceresa cycle via a cycle class map. It was not known to our knowledge whether there exists non-hyperelliptic curves with torsion Ceresa class over characteristic 0 fields. In this talk, I present results from two projects which produce examples of such curves and discuss properties of the Ceresa class.

To make the talk accessible to participants of the summer school, I will explain most of the concepts used in the talk, discuss the relation between the Ceresa class and the Galois action on the geometric fundamental group but omit proofs to the main results. Work presented in the talk are joint with Corey–Ellenberg and Bisogno–Litt–Srinivasan.

RAVI RAMAKRISHNA (Cornell)

Title: The Gras–Munnier Theorem in the tame case
Abstract: Gras–Munnier gave a precise criterion of the existence of a $\mathbb{Z}/p$-extension of a number field K ramified *exactly* at a set of places S. We'll state the theorem and sketch a new proof of it. The original proof involved moderately complicated results from class field theory. Our proof will use one element of class field theory and moderately complicated linear algebra over finite fields.

JEREMY ROUSE (Wake Forest)

Title: 3-adic images of Galois for elliptic curves over $\mathbb{Q}$.
Abstract: We show that if $E/\mathbb{Q}$ is a non-CM elliptic curve over $\mathbb{Q}$, then the 3-adic image of Galois either arises either from one of the genus zero 3-power level modular curves with infinitely many rational points, or is contained in the preimage in $\mathrm{GL}_2(\mathbb{Z}_3)$ of the normalizer of the non-split Cartan

subgroup modulo 27. We compute the tower of arithmetically maximal 3-power level modular curves, compute their equations, and classify the rational points on all of them except $X_{\mathrm{ns}}^{+}(27)$. This is joint work with Andrew Sutherland and David Zureick-Brown.

DAVID SAVITT (Johns Hopkins)

Title: Moduli of Galois representations.
Abstract: In this talk I will explain how the geometry of moduli stacks of two-dimensional Galois representations is related to the weight part of Serre's conjecture for GL(2). This is joint work with Ana Caraiani, Matthew Emerton, and Toby Gee.

BIANCA VIRAY (U. Washington)

Title: Isolated points on curves.
Abstract: Let C be an algebraic curve over $\mathbb{Q}$, i.e., a 1-dimensional complex manifold defined by polynomial equations with rational coefficients. A celebrated result of Faltings implies that all algebraic points on C come in families of bounded degree, with finitely many exceptions. These exceptions are known as isolated points. We explore how these isolated points behave in families of curves and deduce consequences for the arithmetic of elliptic curves. This talk is on joint work with A. Bourdon, Ö. Ejder, Y. Liu, and F. Odumodu. This talk will be suitable for a general audience.

JAN VONK (IAS)

Title: On singular moduli.
Abstract: The theory of complex multiplication occupies an important place in number theory, an early manifestation of which was the use of special values of the j-function in explicit class field theory of imaginary quadratic fields, and the works of Eisenstein, Kronecker, Weber, Hilbert, and many others. In the early 20th century, Hecke studied the diagonal restrictions of Eisenstein series over real quadratic fields, which later lead to highly influential developments in the theory of complex multiplication initiated by Gross and Zagier in their famous work on Heegner points on elliptic curves. In this talk, we will explore what happens when we replace the imaginary quadratic fields in CM theory with real quadratic fields, and propose a framework for a tentative 'RM theory', based on the notion of rigid meromorphic cocycles, introduced in joint work with Henri Darmon. I will discuss several of their arithmetic properties. This concerns various joint works, with Henri Darmon, Alice Pozzi, and Yingkun Li.

TITLES AND ABSTRACTS (Alphabetical by last name) CONTRIBUTED TALKS (LIVE AND PRE-RECORDED)

Santiago Arango Piñeros (Emory) - Pre-recorded (Video)

Title: The Global Field Euler Function.
Abstract: Using a remainder theorem for valuations of a field, we give a new perspective on the norm function of a global field. We define the Euler totient function of a global field and recover the essential analytical properties of the classical arithmetical function, namely the product formula. In addition, we prove the holomorphicity of the associated zeta function. As an application, we recover the analog of the mean value theorem of Erdös, Dressler, and Bateman via the Weiner–Ikehara theorem.

Elisa Bellah (U. Oregon) - Pre-recorded (Video)

Title: Norm Form Equations and Linear Divisibility Sequences.
Abstract: Finding integer solutions to norm form equations is a classic Diophantine problem. Using the units of the associated coefficient ring, we can produce sequences of solutions to these equations. It turns out that such a sequence can be written as a tuple of integer linear recurrence sequences, each with characteristic polynomial equal to the minimal polynomial of our unit. We show that in some cases, these sequences are linear divisibility sequences.

Benjamin Breen (Dartmouth) - Live

Title: Heuristics for narrow class groups of abelian number fields of odd degree.
Abstract: We present heuristics for the behavior of a certain set of ray class fields of a number field that arise when studying the narrow class group. These heuristics allow us to make predictions in the style of Cohen-Lenstra for unit signature ranks and 2-torsion in narrow class groups. We demonstrate our predictions and provide computational support for cyclic fields of degree $n = 3, 5, 7$. This is joint work with Ila Varma and John Voight.

Holly Paige Chaos (U. Vermont) - Live

Title: Torsion for CM Elliptic Curves Defined Over Number Fields of Degree $2p$.
Abstract: Let E be an elliptic curve defined over a number field F. By the Mordell-Weil theorem we know that the points of E with coordinates in F can be given the structure of a finitely generated abelian group. We will focus on the subgroups of points with finite order. For a given prime $p > 3$ and an elliptic curve E defined over a number field of degree $2p$, we would like to know exactly what torsion subgroups arise. Before discussing recent progress on this query, specifically in the case of

elliptic curves with complex multiplication (CM), I will provide a brief overview on elliptic curves as well as outline some significant classical results.

Garen Chiloyan (UConn) - Live

Title: A classification of rational isogeny-torsion graphs.
Abstract: An isogeny graph is a nice visualization of the $\mathbb{Q}$-isogeny class of an elliptic curve defined over $\mathbb{Q}$. A theorem of Kenku shows sharp bounds on the number of distinct isogenies that a rational elliptic curve can have (in particular, every isogeny graph has at most 8 vertices). In this talk, we classify what torsion subgroups over $\mathbb{Q}$ can occur in each vertex of a given isogeny graph of elliptic curves defined over the rationals. This is joint work with Álvaro Lozano-Robledo.

David Corwin (UC Berkeley) - Live

Title: Local-Global Principles for Diophantine Equations and Topology.
Abstract: If a variety has no p-adic points for some prime p, then it has no rational points, and this is a standard way to prove non-existence of rational points. The Hasse or Local-Global Principle asks the converse, whether a variety with p-adic points for every p must have a rational point. Lindt, Reichard, and later Selmer found counterexamples to the Hasse principle in the 1940's, which they showed using reciprocity laws. In 1971, Manin unified their methods into a single "obstruction" to the local-global principle using class field theory, via the Brauer group, and Skorobogatov extended this further in the 90's into what is known as the etale-Brauer obstruction to the local-global principle. In 2009, Poonen constructed a smooth proper variety with no rational points, whose lack of rational points is not explained even by the etale-Brauer obstruction, and since then, Harpaz-Schlank reinterpreted the etale-Brauer obstruction in terms of etale homotopy theory. In this talk, we review the above and then discuss recent work of Schlank and the speaker using the topological perspective of Harpaz-Schlank to better understand Poonen's counterexample. No topology background will be assumed.

Andrew Kobin (U. Virginia) - Live

Title: Stacky curves in characteristic p.
Abstract: As stacks continue to become an essential part of a modern algebraic geometer's toolbox, researchers look to their local structure as a guide to their nature. Over the complex numbers, this local structure is that of a complex orbifold, or "orbit space of a manifold" under a cyclic group action. In this talk, I will survey the classification of stacky curves in characteristic 0 and introduce a new construction, called an Artin–Schreier root stack, which allows for similar classification results in characteristic p.

Debanjana Kundu (U. Toronto) - Live

Title: Iwasawa Theory of Fine Selmer Groups.
Abstract: Classical Iwasawa theory involves studying growth questions of class groups in $\mathbb{Z}_p$-extensions. In 1972, Mazur introduced the Iwasawa theory of Selmer groups of elliptic curves. He described the growth of the p-primary part of the Selmer group in $\mathbb{Z}_p$-towers and showed that in the *ordinary* case the growth is controlled. In 2005, Coates and Sujatha introduced the study of a certain subgroup of the Selmer group, called the fine Selmer group. They showed that these subgroups have stronger finiteness properties than the classical Selmer group.

There is a grand tradition of exploiting the intimate connection between the study of class groups and (fine) Selmer groups of elliptic curves. In this talk, we will discuss some recent results on the growth of fine Selmer groups in p-adic Lie extensions that contribute to strengthening the analogies through which each setting casts light on the other.

Rakvi (Cornell) - Live

Title: On classification of genus 0 modular curves with a rational point
Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$. Fix $\overline{\mathbb{Q}}$ as an algebraic closure of $\mathbb{Q}$. We get a Galois representation

$$\rho_E : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\hat{\mathbb{Z}})$$

by choosing a compatible bases for the N-torsion subgroup of $E(\overline{\mathbb{Q}})$. Let G be an open subgroup of $\mathrm{GL}_2(\hat{\mathbb{Z}})$ such that $\det(G) = \hat{\mathbb{Z}}^\times$ and $-I \in G$. Associated to G we have the modular curve X_G which loosely parametrises elliptic curves E such that $\mathrm{im}(\rho_E) \subseteq G$. In this talk I will discuss my work on the classification of all genus 0 modular curves with a rational point in finitely many families.

Matilde Lalin (Université de Montréal) - Live

Title: The Mahler measure of a genus 3 family.
Abstract: The Mahler measure of a polynomial P is defined as certain integral of $\log|P|$ over the unit torus. For multivariate polynomials, it often yields special values of L-functions. In this talk we will discuss an identity between the Mahler measures of a genus 3 polynomial family and of a genus 1 polynomial family that was initially conjectured by Liu and Qin. This is joint work with Gang Wu.

Caleb Springer (Penn State) - Pre-recorded (Video)

Title: The structure of the group of rational points of an abelian variety over a finite field.
Abstract: The group of rational points of an abelian variety over a finite field can be viewed as a module over the endomorphism ring of the abelian variety. From this viewpoint, Lenstra showed that, in the case of elliptic curves, this module structure has a nice description in terms of the

Frobenius endomorphism. In this talk, I will present a generalization of Lenstra's result to abelian varieties of arbitrary dimension whose endomorphism rings satisfy certain nice properties.

Arvind Suresh (UGA) - Live

Title: Constructing genus g curves of rank $4g + 15$.
Abstract: Let K be a field. A well-known and elementary construction of Mestre produces, for every genus g at least 2, infinitely many genus g curves with $8g + 12$ rational points. Shioda refined this construction to produce infinitely many curves with $8g+16$ rational points, and having Mordell-Weil rank at least $4g + 7$. We show that Mestre's construction can be further refined to produce, for g congruent to 2 mod 3 and exceeding 4, infinitely many curves of genus g having Mordell-Weil rank at least $4g + 15$. For g at least 5, this is a new record for an infinite family of genus g curves of large rank.

Vaidehee Thatte (Binghamton U.) - Live

Title: Upper Ramification Groups for Arbitrary Valuation Rings.
Abstract: We define logarithmic and non-logarithmic ramification filtrations for arbitrary Henselian valuation rings. This joint work with K. Kato generalizes the ramification theory of complete discrete valuation rings of Abbes-Saito. We will discuss the notion of "defect", also known as ramification deficiency, that is specific to arbitrary valuation rings in positive residue characteristic.

Jeffrey Yelton (Emory) - Live

Title: Semistable models of hyperelliptic curves over residue characteristic 2.
Abstract: A celebrated result of Deligne and Mumford says that every curve over a local field attains semistable reduction over a finite algebraic extension of that field. Consider a hyperelliptic curve of the form $y^2 = f(x)$ over a local field K of mixed characteristic (0, 2). We may ask for an explicit method of obtaining a semistable model of this curve over a finite algebraic extension of K and what its special fiber will look like. I will discuss results obtained by my masters student Leonardo Fiore which characterize semistable models of such curves and the geometry of their special fibers according to the distances between the roots of the defining polynomial f, in particular when the genus is 1 or 2.

Joshua Zelinksy (Hopkins School) - Pre-recorded (Video)

Title: Upper bounds in integer complexity.
Abstract: Define $||n||$ to be the *complexity* of n, which is the smallest number of 1s needed to write n using an arbitrary combination of addition and multiplication. John Selfridge showed that

$||n|| \geq 3\log_3 n$ for all n. Richard Guy noted the trivial upper bound that $||n|| \leq 3\log_2 n$ for all $n > 1$ by writing n in base 2. An upper bound for almost all n was provided by Juan Arias de Reyna and Jan Van de Lune. We discuss new work proving an unconditional upper bound. In particular, for all $n > 1$ we have $||n|| \leq \frac{41}{\log 55296} \log n$.