

CTNT 2016 Research Conference
Elliptic Curves, Modular Forms and Related Topics
August 12-14, 2016

ABSTRACTS

Nick Andersen (University of California, Los Angeles)

Kloosterman sums and Maass cusp forms of half-integral weight

ABSTRACT: Kloosterman sums play an important role in modern analytic number theory. I will give a brief survey of what is known about the classical Kloosterman sums and their connection to Maass cusp forms of weight 0. I will then talk about recent progress toward bounding sums of Kloosterman sums of half-integral weight (joint with Scott Ahlgren) where the estimates are uniform in every parameter. Among other things, this requires us to develop a mean value estimate for coefficients of Maass cusp forms of half-integral weight. As an application, we obtain an improved estimate for the classical problem of bounding the size of the error term in Rademacher's formula for the partition function.

John Bergdall (Boston University)

Geometric properties of p -adic families of automorphic forms and applications

ABSTRACT: In this talk we will discuss p -adic families of automorphic forms. Our main topic will be two results on the local geometry of such families: one as ambient rigid analytic space (a smoothness result) and the other relative to the weight map (a non-smoothness result). We will then discuss applications to p -adic arithmetic which depend on the choice of construction of the families.

Luca Candelori (Louisiana State University)

Geometric and p -adic aspects of mock modular forms

ABSTRACT: Based on the geometric theory of p -adic modular forms of Coleman–Katz, we give new geometric interpretations to recent results on p -adic properties of mock modular forms and harmonic Maass forms. We are also able to relate some open questions in the theory to well-known conjectures in algebraic geometry. This is joint work with Francesc Castella (Princeton).

Michael Chou (University of Connecticut)

Torsion of rational elliptic curves over the maximal abelian extension of \mathbb{Q}

ABSTRACT: A theorem of Ribet states that for elliptic curves E/\mathbb{Q} , the size of the torsion subgroup over \mathbb{Q}^{ab} , the maximal abelian extension of \mathbb{Q} , is finite. We will present techniques in order to determine a sharp bound on the size of $E(\mathbb{Q}^{ab})_{\text{tors}}$ for any elliptic curve E/\mathbb{Q} . We further present a list containing all possible torsion subgroups appearing in this way.

Rachel Davis (Purdue University)

Origami Galois representations

ABSTRACT: Let (E, \mathcal{O}) be an elliptic curve defined over \mathbb{Q} . An origami is a pair (C, f) , where C is a curve and $f : C \rightarrow E$ is a map, ramified at most above one point. The name comes from pictures that we will show. We define Galois representations associated to specific origami that generalize the Galois representations arising from the Galois action on the Tate module of E . This is joint work with Edray Goins.

Bill Duke (University of California, Los Angeles)

The distribution of certain surfaces associated to real quadratic fields

ABSTRACT: I will describe joint work with A. Toth and O. Imamoglu. To an ideal class of a real quadratic field we associate a certain surface. This surface, which is a new geometric invariant, has the usual modular closed geodesic as its boundary. Furthermore, its area is determined by the length of an associated backward continued fraction. We study the distribution properties of this surface on average over a genus. In the process we give an extension and refinement of the Katok-Sarnak formula.

Ozlem Ejder (University of Southern California)

Torsion groups of elliptic curves over quadratic cyclotomic fields in elementary abelian 2-extensions

ABSTRACT: Let K denote the quadratic field $\mathbb{Q}(\sqrt{d})$ where $d = -1$ or -3 . Let E be an elliptic curve defined over K . In this paper, we classify the torsion subgroups of E in the maximal elementary abelian 2-extension of K .

Michael Griffin (Princeton University)

Thompson Moonshine

ABSTRACT: The original Monstrous Moonshine conjecture of Conway and Norton embodied many striking apparent coincidences observed by Ogg, McKay, Thompson and others, connecting the then-conjectural Monster group (the largest of the sporadic simple groups) to the theory of modular functions. Borcherds' proof of the conjecture in 1992 included several ideas borrowed from physics. Fifteen years later, Witten brought the theory back to physics, conjecturing a deep connection between Monstrous Moonshine and quantum gravity. Witten's conjecture sparked renewed interest in Moonshine. Prior to its proof, Queen and Norton generalized the original Moonshine conjecture in a natural way to quotient groups of the Monster, including the finite sporadic Thompson group Th . Last year Harvey and Rayhaun conjectured a new moonshine for the Thompson group Th , inherently different from that arising in Generalized Moonshine. In joint work with Michael Mertens, we verify Harvey and Rayhaun's conjecture and prove the existence of an infinite graded Th super-module whose graded traces give the coefficients of the weight $1/2$ modular forms conjectured by Harvey and Rayhaun.

Wade Hindes (The Graduate Center, CUNY)

The average number of integral points in orbits

ABSTRACT: Over a number field K , a celebrated result of Silverman states that if $\phi \in K(x)$ is a rational function whose second iterate is not a polynomial, then the set of integral points in the orbit $\mathcal{O}_\phi(P) = \{\phi^n(P)\}_{n \geq 0}$ is finite for all base points in projective space. In this talk, we study the “average” number of integral points in orbits, and we use this to shed light on certain prime factorization problems in dynamics.

Daeyeol Jeon (Kongju National University)

Modular curves with infinitely many cubic points

ABSTRACT: Modular curves with infinitely many cubic points Abstract: In this talk, we determine all modular curves $X_0(N)$ that admit infinitely many points defined over K when K varies over all cubic number fields.

Edmund Karasiewicz (Rutgers University)

Weyl group multiple Dirichlet series and Fourier coefficients of Eisenstein series on the double cover of $GL(3, \mathbb{R})$

ABSTRACT: The theory of Weyl group multiple Dirichlet series (Dirichlet series in several variables with a group of functional equations isomorphic to a Weyl group) has been developed by Brubaker, Bump, Chinta, et al. These multiple Dirichlet series are conjectured to be the Fourier coefficients of metaplectic Eisenstein series. The work of Brubaker, Bump, Chinta, et al. includes a hypothesis that forces the base field to be totally imaginary. We will consider a specific metaplectic Eisenstein series over \mathbb{R} and see how this fits into the existing theory.

Kiran Kedlaya (University of California, San Diego)

Introducing the L -Functions and Modular Forms Database

ABSTRACT: The L -Functions and Modular Forms Database, officially launched this past spring, is an online resource that aims to compile information about modular and automorphic forms, their L -functions, and the geometric objects related to them (like elliptic curves) in an easily browsable form. I’ll demonstrate some of the existing functionality of the LMFDB as well as some work in progress, and say a little about getting your favorite objects into the database.

Chang Heon Kim (Sungkyunkwan University)

Weakly holomorphic Hecke eigenforms and Hecke eigenpolynomials

ABSTRACT: In the work of Bringmann, Guerzhoy, Kent and Ono weakly holomorphic Hecke eigenforms were defined and constructed in the level one case by making use of harmonic weak Maass forms. In this paper I will extend their results to higher level cases and give an explicit construction in terms of weakly holomorphic modular forms without relying on the theory of harmonic weak Maass forms. Moreover I will find a basis for the space of period polynomials consisting of Hecke eigenpolynomials. This is a joint work with SoYoung Choi.

E. Mehmet Kiral (Texas A&M University)

The Voronoi formula and double Dirichlet series

ABSTRACT: We prove a Voronoi formula for coefficients of a large class of L -functions including Maass cusp forms, Rankin-Selberg convolutions, and certain isobaric sums. Our proof is based on the functional equations of L -functions twisted by Dirichlet characters and does not directly depend on automorphy. Hence it has wider application than previous proofs. The key ingredient is the construction of a double Dirichlet series. This is joint work with Fan Zhou.

Jeffrey Lagarias (University of Michigan)

The Lerch zeta function and the Heisenberg group

ABSTRACT: The Lerch zeta function is a three variable zeta function which generalizes the Riemann zeta function and has a functional equation, but no Euler product. We discuss Hecke operator actions on this function. We give an automorphic interpretation of generalizations of the Lerch zeta function related to the real Heisenberg group modulo a discrete lattice. We discuss adelic extensions. Some work presented is joint work with W.-C. Winnie Li.

Shenhui Liu (Ohio State University)

Central L -derivative values of automorphic forms

ABSTRACT: We will talk about asymptotic formulas for the first and second moments of $L'(1/2, f)$ over an orthogonal Hecke eigenbasis H_{2k} in the space S_{2k} of weight $2k$ cusp forms for the full modular group as odd $k \rightarrow \infty$ with arbitrary power saving. Further, we will talk about how to use mollifiers to establish that there are positive proportion of forms in H_{2k} with nonvanishing central L -derivative value as $k \rightarrow \infty$.

Ling Long (Louisiana State University)

Hypergeometric functions over finite fields

ABSTRACT: Hypergeometric functions are special functions with lot of symmetries. Many combinatorial identities can be derived from hypergeometric functions. In this talk, we will introduce hypergeometric functions over finite fields, originally due to Greene, Katz and McCarthy, in a way that is parallel to the classical hypergeometric functions, and discuss their properties and applications to character sums and the arithmetic of hypergeometric abelian varieties. This is a joint work with Jenny Fuselier, Ravi Ramakrishna, Holly Swisher, and Fang-Ting Tu.

Jeffrey Manning (University of Chicago)

Multiplicities of Galois representations in the mod- ℓ cohomology of Shimura curves

ABSTRACT: In 1990, Ribet observed that it was possible for a Galois representation to occur with multiplicity 2 in the mod- ℓ cohomology of a Shimura curve with discriminant pq and level 1. This result was later reinterpreted by Buzzard, Diamond and Jarvis as part of a conjectural mod- ℓ local-global compatibility statement in the Langlands correspondence for quaternion algebras. We will describe an alternative approach to Ribet's multiplicity 2 result, using a novel application of the Taylor-Wiles-Kisin patching method. This allows us to precisely determine the mod- ℓ multiplicity under much more relaxed the conditions on the discriminant and tame level of the Shimura curve, greatly generalizing Ribet's result. In particular, this allows us to show that a multiplicity of 2^n is achievable for all n . We also present progress towards proving the Buzzard-Diamond-Jarvis conjecture using these techniques.

Christine McMeekin (Cornell University)

On the Distribution of Splitting Behavior in Number Fields Depending on p

ABSTRACT: We construct a number field K_p depending on a fixed number field K and a rational prime p . For certain K , we give distribution results for how p splits in K_p as p varies using results of Friedlander, Iwaniec, Mazur, and Rubin.

Ken Ono (Emory University)

Zeta polynomials for modular forms

ABSTRACT: The speaker will discuss recent work on Manin's theory of zeta polynomials for modular forms. He will describe recent results which confirm Manin's speculation that there is such a theory which arises from periods of newforms. More precisely, for each even weight $k > 2$ newform f the speaker will describe a canonical polynomial $Z_f(s)$ which satisfies a functional equation of the form $Z_f(s) = Z_f(1-s)$, and also satisfies the Riemann Hypothesis: if $Z_f(\rho) = 0$, then $\text{Re}(\rho) = 1/2$. This zeta function is arithmetic in nature in that it encodes the moments of the critical values of $L(f, s)$. This work builds on earlier results of many people on period polynomials of modular forms. This is joint work with Seokho Jin, Wenjun Ma, Larry Rolen, Kannan Soundararajan, and Florian Sprung.

Antonella Perucca (Universität Regensburg)

Reductions of elliptic curves

ABSTRACT: Let E be an elliptic curve defined over a number field K , and fix some prime number ℓ . If $\alpha \in E(K)$ is a point of infinite order, we consider the set of primes \mathfrak{p} of K such that the reduction $(\alpha \bmod \mathfrak{p})$ is well-defined and has order coprime to ℓ . This set admits a natural density. Building on results of R. Jones and J. Rouse (2010) we give two interpretations for this density, and we outline a general strategy for computing it. This is joint work with Davide Lombardo (Paris Orsay).

Rachel Pries (Colorado State University)

Galois action on Fermat curves and Heisenberg extensions

ABSTRACT: Consider the Fermat curve $x^p + y^p = 1$ where p is an odd prime. Let $K = \mathbb{Q}(\zeta_p)$ be the cyclotomic field. We extend work of Anderson about the action of the absolute Galois group G_K on a relative homology group of X . Anderson proved that the action factors through $Q = \text{Gal}(L/K)$ where L is the splitting field of $1 - (1 - x^p)^p$. For p satisfying Vandiver's conjecture, we compute Q and find explicit formula for the action of $q \in Q$ on the relative homology. Using this, we determine the maps between several Galois cohomology groups which arise in connection with obstructions for rational points on the generalized Jacobian. Heisenberg extensions play a key role in the outcome. This is joint work with R. Davis, V. Stojanoska, and K. Wickelgren.

Rufei Ren (University of California, Irvine)

Slopes for higher rank Artin-Schreier-Witt towers

ABSTRACT: We fix a monic polynomial $\bar{f}(x) \in \mathbb{F}_q[x]$ over a finite field of characteristic p , and consider the \mathbb{Z}_{p^l} -Artin-Schreier-Witt tower defined by $\bar{f}(x)$; this is a tower of curves $\cdots \rightarrow C_m \rightarrow C_{m-1} \rightarrow \cdots \rightarrow C_0 = \mathbb{A}^1$, whose Galois group is canonically isomorphic to \mathbb{Z}_{p^l} , the degree l unramified extension of \mathbb{Z}_p , which is abstractly isomorphic to $(\mathbb{Z}_p)^l$ as a topological group. We study the Newton slopes of zeta functions of this tower of curves. This reduces to the study of the Newton slopes of L-functions associated to characters of the Galois group of this tower. We prove that, when the conductor of the character is large enough, the Newton slopes of the L-function asymptotically form a finite union of arithmetic progressions. As a corollary, we prove the spectral halo property of the spectral variety associated to the \mathbb{Z}_{p^l} -Artin-Schreier-Witt tower. This extends the main result in [DWX] from rank one case $l = 1$ to the higher rank case $l \geq 1$.

Holly Swisher (Oregon State University)

Quantum mock modular forms arising from eta-theta functions

ABSTRACT: In 2013, Lemke Oliver classified all eta-quotients which are theta functions. In this work we construct mock modular forms from the eta-theta functions with even characters, such that the shadows of these mock modular forms are given by the eta-theta functions with odd characters. We further prove that the constructed mock modular forms are quantum modular forms. As corollaries, we establish simple finite hypergeometric expressions which may be used to evaluate Eichler integrals of the odd eta-theta functions, as well as some curious algebraic identities. This work is joint with: Amanda Folsom, Sharon Garthwaite, Soon-Yi Kang, and Stephanie Treneer.

Hui Xue (Clemson University)

Degree two monomial relations between eigenforms

ABSTRACT: Duke and Ghate independently answered the question when the product of two eigenforms of level 1 is again an eigenform. In this talk we will investigate the question when the product of two eigenforms of level 1 equals the product of another two eigenforms. Assuming Maeda's conjecture on the irreducibility of Hecke polynomials, we will give some partial answer to the question. As a corollary we also show that eigenforms of level 1 are uniquely determined by their second Fourier coefficients. This is joint work with Trevor Vilardi.

Dong Sung Yoon (KAIST)

Generators of Siegel modular function fields of higher genus and level

ABSTRACT: Siegel pioneered the generalization of the theory of elliptic modular functions to the modular functions in several variables, which are called Siegel modular functions. Siegel modular functions are of fundamental importance in number theory and algebraic geometry. However, we know relatively little about Siegel modular functions until now because it is difficult to find attractive examples that can be handled. In this talk, we construct explicit generators of Siegel modular function field of higher genus and level in terms of multi-variable theta constants.

Fan Zhou (Ohio State University)

Sato-Tate and Plancherel equidistribution of Satake parameters

ABSTRACT: We will talk about two equidistribution results of Satake parameters of automorphic forms on $GL(3)$ over its cuspidal spectrum. The first result is an application of Kuznetsov trace formula on $GL(3)$ of Blomer, Goldfeld-Kontorovich. The second result involves a weight-removing technique to Kuznetsov trace formula. Part of it is a generalization of Shimura's formula for symmetric square L -functions to adjoint representation on $GL(3)$.
