PROJECT: FINDING ELLIPTIC CURVES OF HIGH RANK

The goal of this project is simple: *make history!* More concretely: write your name in the history of elliptic curves with high rank! Even more concretely: write your name in Andrej Dujella's web with the records of elliptic curves with high rank:

https://web.math.pmf.unizg.hr/ duje/tors/tors.html

This is not as simple as it may seem, because many people have been looking for elliptic curves with high rank, and Dujella's website reflects the highest rank records (after many, many hours of work by many mathematicians over many years). Even if you are unable to write your name in history, you may still be able to win a prize in the process!

CONTEST. The student (or team of students) that submits the elliptic curve (over $\mathbb{Q}$) with the highest rank will present their method in the morning of Friday 8/12 and receive a **prize**. *Terms and Conditions:*

(a) The student (or team) must send an email to alvaro@uconn.edu before 11:59pm on Thursday 8/11 with the coefficients of a Weierstrass equation for the curve, and a list of the $\mathbb{Z}$-linearly independent points that prove a lower bound for the rank,

(b) At least, a lower bound of the rank must be verifiable with Magma or Sage, and

(c) The curve must be completely original and not appear in previous literature (the student or team must be able to provide an explanation of how the curve was found). The method, however, can be a combination of methods that do appear in the literature.

Let us begin with a warm-up exercise:

Exercise. The goal here is a systematic way to find curves of rank at least $r \geq 0$ without using tables of elliptic curves:

1. (Easy) Find 3 non-isomorphic elliptic curves over $\mathbb{Q}$ with rank ≥ 2. You must prove that the rank is at least 2. (To show linear independence, you may use PARI or Sage to calculate the height matrix).
2. (Fair) Find 3 non-isomorphic elliptic curves over $\mathbb{Q}$ with rank ≥ 3.
3. (Medium difficulty) Find 3 non-isomorphic elliptic curves over $\mathbb{Q}$ with rank ≥ 6. If so, then you can probably find 3 curves of rank ≥ 8 as well.
4. (Significantly harder) Find 3 non-isomorphic elliptic curves over $\mathbb{Q}$ of rank ≥ 10.
5. (You would be famous!) Find an elliptic curve over $\mathbb{Q}$ of rank ≥ 29.

Project A

The largest rank known to date is 28, due to Noam Elkies. Up to this point, each construction of an elliptic curve (over $\mathbb{Q}$) of high rank follows some *ad-hoc* construction, and there have been many different types of such constructions in the last couple decades. For a list of historical records of ranks, see this website by Dujella. The goal of this project is to familiarize yourself with some of the constructions/references listed in Dujella's pages linked above, and try to replicate the constructions and, if possible, extend them to more general settings. Hopefully, current technology can push the same methods further and find elliptic curves of higher ranks than the original authors were able to at the time the articles were published.

Project B

Read Section 1.3.3 of Garikai Campbell's thesis. In particular, we are interested in the sums $S_E(N)$ and $s_E(N)$ that Mestre and Fermigier first used, and Nagao generalized (and many others have used since). Later, in Section 2.1.2, Campbell defines a new sum $G_E(N)$ and Campbell notes that "it seems to be the case that $(R_E - \lim_{N\to\infty} G_E(N)) < 3/2$". The goal of this project is to study $G_E(N)$ further and, at least, extend computational verification of this inequality to the entirety of Cremona's database, and other databases of elliptic curves.